Geometric measure of entanglement of the qubit with an arbitrary quantum system

Andrzej M. Frydryszak[1], Volodymyr M. Tkachuk[2]

[1)] Institute of Theoretical Physics,
University of Wroclaw, pl. M. Borna 9,
50 - 204 Wroclaw, Poland
e-mail: amfry@ift.uni.wroc.pl

[2)] Department of Theoretical Physics,
Ivan Franko National University of Lviv,
12 Drahomanov St., Lviv, UA-79005, Ukraine
e-mail: tkachuk@ktf.franko.lviv.ua

November 29, 2012

Abstract

We find the explicit expression for geometric measure of entanglement of one qubit with arbitrary other quantum system. The result is applied to the characterization of the bipartite entanglement of the Werner state, Dicke state, GHZ state and trigonometric states for systems consisting of n qubits. The geometric measure of entanglement of one qubit with other $n-1$ qubits in above states is calculated and general properties of bipartite entanglement are established. In particular for the Werner-like states the rule of sums is found and it is shown that deviations from the symmetricity of such states diminishes the amount of entanglement. For Dicke states we show that the maximal value of bipartite entanglement is achieved when number of excitations is half of the total number of qubits in these state. For trigonometric states the bipartite entanglement is maximal and does not depend on the number of qubits.

1 Introduction

Quantum entanglement as the intriguing feature of quantum theory has been noted by Einstein, Podolsky, Rosen, and Shrödinger and Einstein [1, 2] and is intensively studied in the last two decades (see for instance review [3]). As a result it was recognized that entanglement is a new important resource for quantum-information processing.

Quantification of entanglement is one of the principal challenges in quantum information theory. While for a bipartite states the measure of entanglement is well defined, for multipartite states many different measures of entanglement have to be considered [3, 4].

One of the natural entanglement measures is the geometric measure of entanglement, which was proposed by Shimony [5] and generalized for a multipartite system by Wei and Goldbart [6]. It is defined as a minimal squared distance between an entangled state $|\psi\rangle$ and a set of separable states $|\psi_s\rangle$

$$E = \min_{|\psi_s\rangle}(1 - |\langle\psi|\psi_s\rangle|^2) = 1 - \max_{|\psi_s\rangle} |\langle\psi|\psi_s\rangle|^2, \tag{1}$$

where $1 - |\langle\psi|\psi_s\rangle|^2$ is a squared distance of Fubini-Study.

Despite its simple definition it involves a minimization procedure over separable states. Therefore the explicit value of geometric measure of entanglement was derived only for a limited number of entangled states such as GHZ states [6], Dicke states [6, 7], generalized W-states [8], graph states [9] and other types of symmetric states (see also papers [10, 12, 13, 14, 15, 16, 17]).

In this paper we study entanglement of one qubit with some other quantum system which can be, for instance, continuous variable quantum system, arbitrary spin quantum system, composite quantum system, which consist of many qubits or spins. We find in explicit form the geometric measure of entanglement in such case and illustrate the result on some examples.

2 Entanglement of qubit with arbitrary quantum system

Let us consider quantum system which consists of one qubit (or spin 1/2) and some other quantum system. In general, quantum state vector of qubit which is entangled with some other quantum system can be written as follows

$$|\psi\rangle = a|\chi_1\rangle|\phi_1\rangle + b|\chi_2\rangle|\phi_2\rangle, \tag{2}$$

where $|\chi_1\rangle$ and $|\chi_2\rangle$ are two orthogonal vectors which form the basis of one-qubit space; $|\phi_1\rangle$ and $|\phi_2\rangle$ are arbitrary state vectors of quantum system entangled with a qubit, constants a, b are real and positive, phase multipliers can be included into $|\phi_1\rangle$ and $|\phi_2\rangle$, which satisfy normalization conditions $\langle\phi_1|\phi_1\rangle = \langle\phi_2|\phi_2\rangle = 1$. Note that in general this functions are not orthogonal $\langle\phi_1|\phi_2\rangle \neq 0$. Normalization condition $\langle\psi|\psi\rangle = 1$ gives $a^2 + b^2 = 1$.

For beseparable state we have

$$|\psi_s\rangle = |\chi\rangle|\phi\rangle, \tag{3}$$

where

$$|\chi\rangle = \cos(\theta/2)|\chi_1\rangle + \sin(\theta/2)e^{i\alpha}|\chi_2\rangle \tag{4}$$

is an arbitrary qubit state, $|\phi\rangle$ is an arbitrary quantum state of the second system. Note that to describe states of qubit system we use the letter χ with the appropriate index and for the second quantum system we use ϕ.

Then

$$\langle\psi|\psi_s\rangle = a\cos(\theta/2)\langle\phi_1|\phi\rangle + b\sin(\theta/2)e^{i\alpha}\langle\phi_2|\phi\rangle, \tag{5}$$

where $0 \le \theta \le \pi$, $0 \le \alpha \le 2\pi$.

Now we have to find a maximum of $|\langle\psi|\psi_s\rangle|^2$ with respect to $|\phi\rangle$ and with respect to $|\chi\rangle$ which depends on θ and α.

First let us consider the maximum of $|\langle\psi|\psi_s\rangle|^2$ with respect to $|\phi\rangle$. For this purpose let us rewrite (5) as follows

$$\langle\psi|\psi_s\rangle = \lambda\langle\tilde{\phi}|\phi\rangle, \tag{6}$$

where

$$|\tilde{\phi}\rangle = \frac{1}{\lambda}(a\cos(\theta/2)|\phi_1\rangle + b\sin(\theta/2)e^{-i\alpha}|\phi_2\rangle). \tag{7}$$

We find the constant λ from the normalization condition $\langle\tilde{\phi}|\tilde{\phi}\rangle = 1$:

$$\begin{aligned}\lambda^2 &= a^2\cos^2(\theta/2) + b^2\sin^2(\theta/2) + 2ab\sin(\theta/2)\cos(\theta/2)\cos(\alpha-\beta) \qquad (8)\\ &\cdot|\langle\phi_1|\phi_2\rangle| = \frac{1}{2} + \frac{1}{2}(a^2-b^2)\cos(\theta) + ab\sin(\theta)\cos(\alpha-\beta)|\langle\phi_1|\phi_2\rangle|,\end{aligned}$$

where $\langle\phi_1|\phi_2\rangle = |\langle\phi_1|\phi_2\rangle|e^{i\beta}$, and thus

$$|\langle\psi|\psi_s\rangle|^2 = \lambda^2|\langle\tilde{\phi}|\phi\rangle|^2. \tag{9}$$

The maximum of $|\langle\psi|\psi_s\rangle|^2$ with respect to $|\phi\rangle$ is achieved when $|\phi\rangle = |\tilde{\phi}\rangle$:

$$\max_{|\phi\rangle} |\langle\psi|\psi_s\rangle|^2 = \lambda^2 = \frac{1}{2} + \frac{1}{2}(a^2 - b^2)\cos(\theta) + \tag{10}$$

$$ab\sin(\theta)\cos(\alpha - \beta)|\langle\phi_1|\phi_2\rangle|. \tag{11}$$

The maximum with respect to α is achieved at $\alpha = \beta$. Then

$$\max_{|\phi\rangle,\alpha} |\langle\psi|\psi_s\rangle|^2 = \frac{1}{2} + \frac{1}{2}(a^2 - b^2)\cos(\theta) + ab\sin(\theta)|\langle\phi_1|\phi_2\rangle|. \tag{12}$$

In order to find the maximum of this expression with respect to θ let us make a simple trigonometric transformation, namely

$$\max_{|\phi\rangle,\alpha} |\langle\psi|\psi_s\rangle|^2 = \frac{1}{2} + \frac{1}{2}\sqrt{(a^2 - b^2)^2 + 4a^2b^2|\langle\phi_1|\phi_2\rangle|^2}\cos(\theta - \theta'), \tag{13}$$

where

$$\tan(\theta') = \frac{2ab|\langle\phi_1|\phi_2\rangle|}{a^2 - b^2}. \tag{14}$$

As we see maximum of (13) is achieved at $\theta = \theta'$ and we have

$$\max_{|\phi\rangle,\alpha,\theta} |\langle\psi|\psi_s\rangle|^2 = \frac{1}{2} + \frac{1}{2}\sqrt{(a^2 - b^2)^2 + 4a^2b^2|\langle\phi_1|\phi_2\rangle|^2}. \tag{15}$$

Thus, the geometric measure of bipartite entanglement reads

$$E = \frac{1}{2}\left(1 - \sqrt{(a^2 - b^2)^2 + 4a^2b^2|\langle\phi_1|\phi_2\rangle|^2}\right). \tag{16}$$

Note that the maximal value of geometric measure of entanglement $E_{\max} = 1/2$ is achieved at $\langle\phi|\phi\rangle = 1$ and $a = b$. Thus $0 \leq E \leq 1/2$.

Let us apply the general result (16) to some specific cases.

Example 1. Consider $|\phi_1\rangle = |\phi_2\rangle$. In this case state (2) is separable. Then

$$E = \frac{1}{2}\left(1 - \sqrt{(a^2 - b^2)^2 + 4a^2b^2}\right) = \frac{1}{2}\left(1 - \sqrt{(a^2 + b^2)^2}\right) = 0, \tag{17}$$

as it must be in this case. Here we use the normalization condition $a^2 + b^2 = 1$.

Example 2 Let the second system consists of one qubit. Then the Schmidt decomposition reads

$$|\psi\rangle = a|\chi_1\rangle|\phi_1\rangle + b|\chi_2\rangle|\phi_2\rangle, \tag{18}$$

where $\langle\chi_1|\chi_2\rangle = 0.$ and $\langle\phi_1|\phi_2\rangle = 0$. Then

$$E = \frac{1}{2}\left(1 - |(a^2 - b^2)|\right) \tag{19}$$

and for $a \geq b$

$$E = \frac{1}{2}\left(1 - (a^2 - b^2)\right) = b^2. \tag{20}$$

From the other hand, for two entangled qubits (spins) the concurrence reads

$$C = 2ab = 2b\sqrt{1 - b^2}. \tag{21}$$

So

$$b^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - C^2}. \tag{22}$$

As a result of condition $a \geq b$ we find

$$b^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - C^2}. \tag{23}$$

Thus

$$E = \frac{1}{2}(1 - \sqrt{1 - C^2}). \tag{24}$$

This special case reproduces the well known result.

3 Geometric measure of entanglement for principal families of n-qubit states

Entanglement monotones for multi-qubit systems are still to be defined for higher n, $n > 4$, even for pure states. Here we want to get some partial information on the measure of entanglement for some families of n-qubit pure states known to be highly entangled for lower n. Namely, we shall consider n-qubit Werner states, Dicke states, GHZ states and trigonometric states (n-qubit cosine and sine states). An interesting conclusion is that for some of above families degree of bipartite entanglement does not depend on the size of the system i.e. number of qubits.

3.1 Werner states

In general the Werner-like state (generalized W-state) for n qubits reads

$$|W_n\rangle = c_1|100...0\rangle + c_2|010...0\rangle + ... + c_n|000...1\rangle, \tag{25}$$

where $\sum_i |c_i| = 1$. We can write it in the following form

$$|W_n\rangle = c_1|1\rangle_1|00...0\rangle_{2...n} + |0\rangle_1(c_2|10...0\rangle_{2...n} + ... + c_n|00...1\rangle_{2...n}). \tag{26}$$

It can be reduced to form (2), where

$$\begin{aligned} |\chi_1\rangle &= |1\rangle_1, \quad |\chi_2\rangle = |0\rangle_1, && (27)\\ |\phi_1\rangle &= |0\rangle_2...|0\rangle_n, && (28)\\ |\phi_2\rangle &= \frac{1}{|c_2|^2 + ...|c_n|^2}(c_2|10...0\rangle_{2...n} + ... + c_n|00...1\rangle_{2...n}), && (29) \end{aligned}$$

and

$$a = |c_1|, \quad b^2 = |c_2|^2 + ...|c_n|^2 = 1 - |c_1|^2. \tag{30}$$

Note that for above decoposition $\langle\phi_1|\phi_2\rangle = 0$. According to Eq. (16) the entanglement of the first qubit with $n-1$ other qubits in Werner state is

$$E_1 = \frac{1}{2}(1 - |a^2 - b^2|) = \frac{1}{2}(1 - |1 - 2|c_1|^2|). \tag{31}$$

Obviously, for entanglement of the i-th qubit with other qubits we have the similar result

$$E_i = \frac{1}{2}(1 - |1 - 2|c_i|^2|). \tag{32}$$

Note, that maximal value $E_i = 1/2$ is attained at $|c_i|^2 = 1/2$.

In conclusion let us note an interesting relation. Consider such a Werner-like state for which all $|c_i|^2 \leq 1/2$. Then

$$E_i = |c_i|^2 \tag{33}$$

and thus

$$\sum_{i=1}^{n} E_i = \sum_{i=1}^{n} |c_i|^2 = 1. \tag{34}$$

In particular, above sum roule is valid for the proper Werner state of n-qubits with $c_i = \frac{1}{\sqrt{n}}$ and partial entanglement measures $E_i = \frac{1}{n}$. The

small deviations from symmetricity of the Werner-like state (i.e. such that c_i are not equall, but still $|c_i|^2 \leq \frac{1}{2}$) do not change the total amount of entanglement. When not all c_i are such that $|c_i|^2 \leq 1/2$, then we get only the majorization. Namely, let $|c_m|^2 > 1/2$. Obviously, it is only possible for one value of the index m, hence

$$\sum_{i=1}^{n} E_i = \sum_{i=1, i\neq m}^{n} |c_i|^2 + 1 - |c_m|^2 = 2(1 - |c_m|^2) < 1. \tag{35}$$

This means that for a strongly nonsymmetric Werner-like states i.e. with one coefficient c_m such that $1 > |c_m|^2 > \frac{1}{2} \geq |c_i|^2$ the amount of total entanglement is diminished and $\sum E_i \to 0$ for $|c_m|^2 \to 1$.

3.2 Dicke states

The Dicke state for n qubits is defined as follows

$$\begin{aligned} |D_{n,k}\rangle = A \sum_{perm} \underbrace{|0\rangle \ldots |0\rangle}_{n-k} \underbrace{|1\rangle \ldots |1\rangle}_{k} = \qquad (36) \\ = A(|1,1,...,1,1,0,0,...,0\rangle + |1,1,...,1,0,1,0,...,0\rangle + ..., \\ +|0,0,...,0,1,1,...,1\rangle). \end{aligned}$$

Each state in this superposition contains k unities (excitations) and $n-k$ zeros. the number of states in (36) is $C_n^k = n!/k!(n-k)!$ and thus the normalization constant $A = \sqrt{1/C_n^k}$. Distinguishing the first qubit we rewrite (36) in the form (2) where

$$\begin{aligned} |\chi_1\rangle = |1\rangle_1, \quad |\chi_2\rangle = |0\rangle_1, \qquad (37) \\ |\phi_1\rangle = |D_{n-1,k-1}\rangle_{2,...,n} = A_1 \sum_{perm} \underbrace{|0\rangle \ldots |0\rangle}_{n-k} \underbrace{|1\rangle \ldots |1\rangle}_{k-1} \\ |\phi_2\rangle = |D_{n-1,k}\rangle_{2,...,n} = A_2 \sum_{perm} \underbrace{|0\rangle \ldots |0\rangle}_{n-k-1} \underbrace{|1\rangle \ldots |1\rangle}_{k}. \end{aligned}$$

Note that states in $|\phi_1\rangle$ contain $k-1$ units and $n-k$ zeros, states in $|\phi_2\rangle$ contain k units and $n-k-1$ zeros. The number of states in $|\phi_1\rangle$ is $C_{n-1}^{k-1} = (n-1)!/(k-1)!(n-k)!$ and in $|\phi_2\rangle$ is $C_{n-1}^{k} = (n-1)!/(k)!(n-1-k)!$. Normalization constants are $A_1 = 1/\sqrt{C_{n-1}^{k-1}}$ and $A_2 = 1/\sqrt{C_{n-1}^{k}}$. Then $a = A/A_1 = \sqrt{C_{n-1}^{k-1}/C_n^k} = \sqrt{n_1/n}$ and $b = \sqrt{C_{n-1}^{k}/C_n^k} = \sqrt{(n-k)/n}$.

Taking into account the orthogonality of states $|\phi_1\rangle$ and $|\phi_2\rangle$ the entanglement measure of one qubit with $n-1$ other qubits reads

$$E_1 = \frac{1}{2}(1 - |a^2 - b^2|) = \frac{1}{2}(1 - |1 - \frac{2k}{n}|). \tag{38}$$

The maximal value of the measure of entanglement $E_1 = 1/2$ is reached at $k = n/2$.

Note that for $k = 1$ the Dicke state is, in fact, equall to the proper Werner $|D_{n,1}\rangle = |W_n\rangle$ and for the entanglement mesure we obtain $E_1 = \frac{1}{n}$. This is in agrement with the result obtained in the previous section for proper Werner state. For $k > 1$, $|D_{n,k}\rangle$ are also called the cluster Werner states. The states $|D_{n,k}\rangle$ and $|D_{n,n-k}\rangle$ are dual in the sense of the general notion of pure state duality introduced in Ref. [18, 19].

3.3 GHZ states

A general GHZ-like state can be taken in the form

$$|GHZ\rangle = c_1|000...0\rangle + c_2|111...1\rangle \tag{39}$$

in this case

$$|\chi_1\rangle = |0\rangle_1, \quad |\chi_2\rangle = |1\rangle_1, \tag{40}$$
$$|\phi_1\rangle = |00...0\rangle_{2...n}, \tag{41}$$
$$|\phi_2\rangle = |11...1\rangle_{2...n}, \tag{42}$$

with $a = |c_1|$ and $b = |c_2| = \sqrt{1 - |c_1|^2}$. As for the Werner states above $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthogonal, but contrary to the Werner case the number of constants c_1 and c_2 is independent on the size of the n-qubit system. Hence

$$E = \frac{1}{2}(1 - |1 - 2|c_1|^2|). \tag{43}$$

For the proper GHZ-state $E = 1/2$.

3.4 Trigonometric states

Recently there were discussed states with interesting properties which in the formalism of nilpotent quantum mechanics [18, 19] are naturally defined as trigonometric functions of nilpotent commuting variables [20, 21]. The formalism using η-variables (commuting nilpotent variables) is very efficient

in the algebraical description of entanglement and reveals many properties of multi-qubit states from the functional point of view. As concerns η-trigonometric functions, isolated examples of states belonging to these families were considered independently of this context in quantum optics, as pure states with interesting entanglement properties appropriate to test entanglement monotones [22]. Here we shall consider the *sin*-states and *cos*-states using their binary basis representation only (the η-function representation is given in Refs. [18, 19, 20, 21] where relation to trigonometric functions is shown).

To get some intuition let us begin with some explicit formulas for $n = 3, 4$. We shall use the following naming convention: trigonometric state is a state of n-qubits defined by η-trigonometric function and normalized. The cosine and sine states for three qubits have the following form

$$|\psi_c^{(3)}\rangle = \frac{1}{2}(|000\rangle - |110\rangle - |101\rangle - |011\rangle), \quad (44)$$

$$|\psi_s^{(3)}\rangle = \frac{1}{2}(|100\rangle + |010\rangle + |001\rangle - |111\rangle). \quad (45)$$

Now the one of possible bipartite decompositions of such a states can be written as

$$|\psi_c^{(3)}\rangle = \frac{1}{2}|0\rangle(|00\rangle - |11\rangle) - \frac{1}{2}|1\rangle(|10\rangle + |01\rangle), \quad (46)$$

$$|\psi_s^{(3)}\rangle = \frac{1}{2}|0\rangle(|10\rangle + |01\rangle) + \frac{1}{2}|1\rangle|00\rangle - |11\rangle). \quad (47)$$

Identifying two-qubit GHZ and W states $|GHZ\rangle^- = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ and $|W\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ we can write

$$|\psi_c^{(3)}\rangle = \frac{1}{\sqrt{2}}|0\rangle|GHZ\rangle^- - \frac{1}{\sqrt{2}}|1\rangle|W\rangle, \quad (48)$$

$$|\psi_s^{(3)}\rangle = \frac{1}{\sqrt{2}}|0\rangle|W\rangle + \frac{1}{\sqrt{2}}|1\rangle|GHZ\rangle^-. \quad (49)$$

As $|W\rangle$ and $|GHZ\rangle^-$ are orthogonal from the Eq. (16) one gets that $E(\psi_c^{(3)}) = E(\psi_s^{(3)}) = \frac{1}{2}$.

To illustrate the form of cosine and sine states for even number of qubits let us write them explicitely for four qubits. Namely, they take the following

form

$$|\psi_c^{(4)}\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle - |1100\rangle - |1010\rangle - |1001\rangle - |0110\rangle \tag{50}$$
$$- |0101\rangle - |0011\rangle + |1111\rangle),$$
$$|\psi_s^{(4)}\rangle = \frac{1}{2\sqrt{2}}(|1000\rangle + |0100\rangle + |0010\rangle + |0001\rangle - |1110\rangle \tag{51}$$
$$- |1101\rangle - |1011\rangle - |0111\rangle).$$

Now, we shall consider generic families of trigonometric states $|\psi_s^{(n)}\rangle$ and $|\psi_c^{(n)}\rangle$ for n qubits, with arbitrary n. The motivation of the definition comes from the η-function formalism and can be found in [18, 19]. Let $|cos\alpha_n\rangle$ and $|sin\alpha_n\rangle$ denotes cosine and sine for n qubits, using binary bases we have

$$|sin\alpha_n\rangle = \sum_{k\ odd}\sum_{perm}(-1)^{\frac{k-1}{2}}\underbrace{|0\rangle\ldots|0\rangle}_{n-k}\underbrace{|1\rangle\ldots|1\rangle}_{k} \tag{52}$$

and similarly

$$|cos\alpha_n\rangle = \sum_{k\ even}\sum_{perm}(-1)^{\frac{k}{2}}\underbrace{|0\rangle\ldots|0\rangle}_{n-k}\underbrace{|1\rangle\ldots|1\rangle}_{k}. \tag{53}$$

Resulting normalized states have the following form [21]

$$|\psi_s^{(n)}\rangle = 2^{-(\frac{n-1}{2})}|sin\alpha_n\rangle, \tag{54}$$
$$|\psi_c^{(n)}\rangle = 2^{-(\frac{n-1}{2})}|cos\alpha_n\rangle. \tag{55}$$

To calculate geometric measure of entanglement using Eq. (16) let us write above states in bipartite decomposition into the subsystems composed of one qubit and $n-1$ qubits. Using reduction formulas for the "sum of angles" for η-trigonometric functions [18] we can write decompositions (we detach one qubit from the rest $n-1$ qubits)

$$|sin\alpha_n\rangle = |0\rangle|sin\alpha_{n-1}\rangle + |1\rangle|cos\alpha_{n-1}\rangle, \tag{56}$$
$$|cos\alpha_n\rangle = |0\rangle|cos\alpha_{n-1}\rangle - |1\rangle|sin\alpha_{n-1}\rangle. \tag{57}$$
$$\tag{58}$$

above relations written in terms of normalized states give

$$|\psi_s^{(n)}\rangle = 2^{-\frac{1}{2}}(|0\rangle|\psi_s^{(n-1)}\rangle + |1\rangle|\psi_c^{(n-1)}\rangle), \tag{59}$$
$$|\psi_c^{(n)}\rangle = 2^{-\frac{1}{2}}(|0\rangle|\psi_c^{(n-1)}\rangle - |1\rangle|\psi_s^{(n-1)}\rangle). \tag{60}$$

Finally, the geometric entanglement measure for above families of states gives n independent value i.e. $E(|\psi_s^n\rangle) = E(|\psi_c^n\rangle) = \frac{1}{2}$. It is interesting because, as we have seen for $n = 3$, the trigonometric states can be decomposed into an appropriate sum containing Werner or cluster Werner states (i.e.$|D_{n,k}\rangle$), for which the value of entanglement measure depends on n, but these states enter the trigonometric state expansion in such a way that the total result is independent of the number of qubits.

4 Conclusions

In this paper we give explicit formula (16) for the geometric measure of entanglement between one qubit and arbitrary quantum system. This is a general result which can be applied to various quantum states. As an example we consider the measure of entanglement of a one qubit with the $n-1$ qubits in the n-qubit Werner states, Dicke states, GHZ states and trigonometric states (n-qubit cosine and sine states). In the case of Werner state we establish the rule of sum for entanglement. Namely, when all $|c_i|^2 \leq 1/2$, the sum of geometric measure of entanglements of all qubits is equal 1 (almost symmetric Werner-like states) and for the nonsymmetric Werner-like states the degree of entanglement is diminished. For the Dicke states we show that the maximal value of the entanglement of one qubit with the rest of the system is achieved when numbers of units (excitations) equals to number of zeroes. Moreover, for trigonometric states the measure of entanglement of a qubit with remaining ones is maximal and does not depend on the number of qubits.

References

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[2] E. Schrödinger, Naturwiss. **23**, 807 (1935); **23**, 823 (1935); **23**, 844 (1935).

[3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).

[4] M. B. Plenio, S. Virmani, Quantum Inf. Comp. **7**, 1 (2007).

[5] A. Shimony, Ann. N. Y. Acad. Sci. **755**, 675 (1995).

[6] T. C. Wei, P. M. Goldbart, Phys. Rev. A **68**, 042307 (2003).

[7] J. Martin, O. Giraud, P. A. Braun, D. Braun, T. Bastin, Phys. Rev. A **81**, 062347 (2010).

[8] Sayatnova Tamaryan, Anthony Sudbery, Levon Tamaryan, Phys. Rev. A 81, 052319 (2010).

[9] Damian Markham, Akimasa Miyake, Shashank Virmani, New J. Phys. **9**, 194 (2007).

[10] Levon Tamaryan, Dae Kil Park, Sayatnova Tamaryan, Phys. Rev. A 77, 022325 (2008).

[11] Levon Tamaryan, Dae Kil Park, Jin-Woo Son, Sayatnova Tamaryan, Phys. Rev. A 78, 032304 (2008).

[12] Ali Saif M. Hassan, Pramond S. Jong, Phys. Rev. A **77**, 062334 (2008).

[13] Masahito Hayashi, Damian Markham, Mio Murao, Masaki Owari, Shashank Virmani, J. of Math. Phys., **50**, 122104 (2009).

[14] Levon Tamaryan, Zaruhi Ohanyan, Sayatnova Tamaryan, Phys. Rev. A **82** 022309 (2010).

[15] Lin Chen, Aimin Xu, H Ugjun Zhu, Phys. Rev. A **62** 032301 (2010).

[16] Alexander Streltsov, Hermann Kampermann, Dagmar Bruss, Phys. Rev. A **84**, 022323 (2011).

[17] Damian J. H. Markham, Phys. Rev. A83, 042332 (2011).

[18] Andrzej M. Frydryszak, "Nilpotent quantum mechanics, qubits, and flavors of entanglement", arXiv:0810.3016 [quant-ph].

[19] Andrzej M. Frydryszak, Int. J. Mod. Phys. **A 22**, 2513–2533 (2007).

[20] Andrzej M. Frydryszak, "η-trigonometric states of four qubits and entanglement measures", arXiv:0902.3553v1 [quant-ph].

[21] Andrzej M. Frydryszak, Int. J. Geom. Methods Mod. Phys. **9**, 1261005 (2012)

[22] X-J. Ren, W. Jiang, X. Zhou, Z-W. Zhou, and G-C. Guo, Phys. Rev. A **78**, 012343 (2008).